International Journal of Scientific & Engineering Research Volume 8, Issue 10, October-2017
ISSN 2229-5518

A Note on Vorticity Of Unsteady Mhd Free Convective Flow And Mass Transfer Through Porous Medium With Radiation And Variable Permeability In Slip Flow Regime

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ABSTRACT

This paper deals with the study of the vorticity of unsteady hydromagnetic free convective flow and mass transfer through a viscous incompressible and electrically conducting fluid through a porous medium with radiation and variable permeability in slip flow regime. The vorticity of the flow has been found for different values of Magnetic parameter (M), Grashof number (G_r), Modified Grashof number (G_m) ,Prandle number (P_r) , Schmidt number (S_c) and Radition parameter(R).

Keywords: Hydromagnetic flow, Porous medium, Vorticity, Free convective Flow

1 INTRODUCTION

In recent years, there has been a considerable interest in rotating hydromagnetic fluid flows due to possible applications to geophysical and astrophysical problems. An order of magnitude analysis shows that in the basic field equations, the Coriolis force is very significant as compared to the inertial force. Furthermore, it reveals that the Coriolis and magnetohydrodynamic forces are of comparable magnitude. It is generally admitted that a number of astronomical bodies (e.g. the Sun, Earth, Jupiter, magnetic Stars, pulsars) possess fluid interiors and (at least surface) magnetic fields. Changes in the rotation rate of such objects suggest the possible importance of hydromagnetic spin-up. This problem of spin-up in magnetohydrodynamic rotating fluids has been examined under varied conditions by many researchers notably Gilman and Benton [8], Benton and Loper [2], Chawala [5], Debnath [6,7] and Singh [14]. In all these analyses, the effect of the Hall current is not considered. Kim [10] investigated the heat and mass transfer in MHD micrpoloar flow over a vertical moving porous plate in a porous medium. The problem of electromagnetic free convection flow of a micropolar fluid with relaxation time through a porous medium was discussed by Zakaria [18]. Kim [9] studied the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, when the induced magnetic field and viscous dissipation are not taken into account. He found that increase magnetic parameter decreases velocity. The unsteady MHD flow of electrically conducting viscous incompressible non-Newtonian Bingham fluids bounded by two parallel non conducting porous plates with heat transfer considering the Hall Effect has been studied by Attia and Ahmed [1]. Borkakati and Bharali [3] have heat transfer between two infinite horizontal parallel porous plates, where the lower plate is a stretching sheet and the upper one is a porous solid plate in presence of a transverse magnetic field. The heat transfer in an axisymmetric flow between two parallel porous disks under the effect of a transverse magnetic field was studied by Borkakati and Bharali [4]. Saeid [13] presented an analysis of mixed convection in a vertical porous layer using non–equilibrium model. Periodic free convection from a vertical plate in a saturated porous medium, non–equilibrium model was given by Saeid and Abdulmajeed [13].The problem of combined free and forced convective magneto hydrodynamic flow in a vertical channel has been studied by Umavathi and Malashetty [17]. They had also considered the effect of viscous and ohm dissipations. It has been observed that the viscous dissipation enhances the flow reversal in the case of downward flow while it countered the flow in the case of upward flow. Kumari, M. and Kumar, M. [11] discussed an unsteady free convection flow through two vertical plates in presence of uniform magnetic field.

Singha [16] investigated the effect of heat transfer on unsteady hydromagnetic flow in a parallel plates channel of an electrically conducting, viscous, incompressible fluid. He found the velocity distribution increases near the plates and then decrease very slowly at the central portion between the two plates. Very recently, Singha *et al.* [15] investigated the unsteady two- dimensional magnetohydrodynamic Couette flow and heat transfer of an electrically conducting, viscous, incompressible fluid bounded by two plane non-conducting parallel plates placed horizontally is studied in the presence of a uniform transfer magnetic field.

In this paper we analyzed the vorticity of unsteady hydromagnetic free convective flow and mass transfer through porous medium with radiation and variable permeability in slip flow regime. The vorticity has been analyzed for variations in the different parameters involved in the problem.

2 FORMULATION OF THE PROBLEM:

We consider two dimensional unsteady hydromagnetic free convection with radiation and mass transfer flow of a viscous incompressible and electrically conducting fluid through a porous medium of variable permeability occupying semi-infinite region of space bounded by an infinite vertical porous plate with constant suction in slip flow regime. We take x-axis along the plate and y-axis is taken normal to it.

Under these condition the flow equation can be written as

Equation of Continuity:

$$\frac{\partial v}{\partial y} = 0 \qquad \dots (1.1)$$

Equation of Momentum:

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + v\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_0^2 u}{\rho} - \frac{\upsilon u}{K(t)} \qquad \dots (1.2)$$

3 METHOD OF SOLUTION

The equation of continuity (1.1) gives

$$v = -v_0 = \text{Constant} \qquad \dots (1.3)$$

Where $v_0 > 0$ corresponds to the constant suction velocity at the plate. In view of equation (1.3) equation (1.1) and (1.2) can be written as:

$$\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + v\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_0^2 u}{\rho} - \frac{\upsilon u}{K(t)} \qquad (1.4)$$

The permeability of the porous medium is assumed to be the form

$$k(t) = K_0(1 + \varepsilon e^{-nt}) \qquad \dots (1.5)$$

Where K_0 is the mean permeability of the medium, n is the real constant, t is the time and ε (<<1) is a constant quantity.

The boundary conditions are

International Journal of Scientific & Engineering Research Volume 8, Issue 10, October-2017
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$$u = L_1\left[\frac{\partial u}{\partial y}\right], C = C_w, T = T_w \text{ at y=0}$$

$$u = 0, C \to C_w, T \to T_w \text{ as } y \to \infty$$

… (1.6)

Let us introduce the following non dimensional quantities

$$u^* = \frac{u}{v_0},\ y^* = \frac{v_0 y}{\upsilon},\ P_r = \frac{\mu C_p}{K},\ S_C = \frac{\upsilon}{D},$$

$$t^* = \frac{v_0^2 t}{4\upsilon},\ \phi = \frac{C - C_\infty}{C_w - C_\infty}, \theta = \frac{T - T_\infty}{T_w - T_\infty},\ K_0^* = \frac{v_0^2 K_0}{\upsilon^2}$$

$$M^2 = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2},\ h_1 = \frac{L_1 v_0}{\upsilon}, R = \frac{4\upsilon l}{\rho C_p v_0^2},$$

$$A = \frac{D_1(T_w - T_\infty)}{\upsilon(C_w - C_\infty)}\ G_r = \frac{\upsilon g \beta (T_w - T_\infty)}{\upsilon_0^3},$$

$$G_m = g\beta^* \frac{m\upsilon^2}{V_0^4 D}$$

The equation (1.4) in view of (1.5) in non dimensional form after dropping the stars over the primed quantities, we get

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G_r\theta + G_m\phi + \frac{\partial^2 u}{\partial y^2} - \left[M^2 + \frac{1}{K_0(1 + \varepsilon e^{-nt})}\right.$$

… (1.7)

With corresponding boundary conditions:

$$u = h_1 \frac{\partial u}{\partial y},\quad \theta = 1, \phi = 1 \text{ at y=0}$$

$$u \to 0,\quad \theta \to 0, \phi \to 0 \text{ as } y \to \infty$$

… (1.8)

The partial differential equation (1.7) is reduced to ordinary one by assuming the following expression for the velocity

$$u(y,t) = u_0(y) + \varepsilon e^{-nt} u_1(y)$$

… (1.9)

Substituting equation (1.9) in equation (1.7) and equating the coefficients of the powers of ε , the following set of ordinary differential equation are obtained;

$$u_0'' + u_0' - \left[M^2 + \frac{1}{K_0}\right]u_0 = -G_r\theta_0 - G_m\phi_0$$

… (2.0)

$$u_0'' + u_0' - \left[M^2 + \frac{1}{K_0} - \frac{n}{4}\right]u_1 = -G_r\theta_1 - G_m\phi_1 - \frac{u_0}{K_0}$$

… (2.1)

With corresponding boundary conditions;

$$u_0 = h_1 u_0',\ u_1 = h_1 u_0', \theta_0 = 1, \theta_1 = 0, \phi_0 = 1,$$

$\phi_1 = 0$ at y=0

$$u_0 \to 0, u_1 \to 0, \theta \to 0, \theta_1 \to 0, \phi_0 \to 0, \phi_1 \to 1$$

as $y \to \infty$ … (2.2)

Solving the equation (2.0) and (2.1) using the boundary conditions (2.2), we obtain

$$u(y,t) = \left[d_1 e^{-a_2 y} + a_3(G_m d_1 - G_r)e^{-a_1 y} - G_m a_4(1 + d_1)e^{-S_c y}\right] + \frac{\varepsilon e^{-\delta t}}{K_0}\left[d_3 e\right.$$

... (2.3)

$$\zeta = \left[-a_2 d_1 e^{-a_2 y} - a_1 a_3(G_m d_1 - G_r)e^{-a_1 y} + S_c G_m a_4(1 + d_1)e^{-S_c y}\right] + $$

... (2.4)

Where,

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$$a_1 = \frac{P_r + \sqrt{P_r + 4RP_r}}{2},\ a_2 = \frac{1+\sqrt{4(M^2 + \frac{1}{k_0})}}{2},$$

$$a_3 = \frac{1}{a_1^2 - a_1 - \left(M^2 + \frac{1}{K_0}\right)}$$

$$a_4 = \frac{1}{s_c^2 - s_c - \left(M^2 + \frac{1}{K_0}\right)},$$

$$a_5 = 1 + \frac{\sqrt{1+4(M^2 + \frac{1}{k_0} - \frac{\delta}{4})}}{2},$$

$$b_1 = \frac{1}{a_2^2 - a_1 - \left(M^2 + \frac{1}{K_0} - \frac{\delta}{4}\right)},$$

$$b_2 = \frac{1}{a_1^2 - a_1 - \left(M^2 + \frac{1}{K_0} - \frac{\delta}{4}\right)}$$

$$b_3 = \frac{1}{s_c^2 - s_c - \left(M^2 + \frac{1}{K_0} - \frac{\delta}{4}\right)}, \qquad d_1 = \frac{a_1 s_c a}{a_1 - s_c},$$

$$d_2 = \frac{(G_r - G_m d_1)a_3\left(1+h_1 a_1\right) + G_m a_4 (1+d_1)\left(1+h_1 s_c\right)}{(1+h_1 a_2)}$$

$$d_3 = \frac{(1+h_1 a_2)d_4 + (d_1 G_m - G_r)(1+h_1 a_1)d_5 - \left(1+h_1 s_c\right)}{(1+h_1 a_5)}$$

$$,\ d_4 = b_1 d_2,\ d_5 = b_2 a_3,\ d_6 = b_3 a_4\left(1+d_1\right)$$

4. RESULT AND DISCUSSION

Table 1 Vorticity profile for different values of M and $G_m = 2$, G_r=2, M=2, $h_1 = 0.8$, $R = 1$,

$\varepsilon = 0.01$, $\delta = 1$, $S_c = 0.6$, $K_0 = 1.0$

Y	0	1	2	3	4	5
M=0	17.8127	3.4120	0.1528	-0.3840	-0.3387	-0.2210
M=0.5	9.9296	1.7539	-0.0050	-0.2605	-0.2126	-0.1356
M=1.0	4.9587	0.8430	0.0091	-0.1115	-0.0923	-0.0592

Table 2 Vorticity profile for different values of P_r and $G_m = 2$, $G_r = 2$, $h_1 = 0.8$, $R = 1$, $\varepsilon = 0.01$,

$\delta = 1$, $M = 2$, $S_c = 0.6$, $K_0 = 1.0$

Y	0	1	2	3	4	5
P_r =0.71	3.1461	0.6474	0.1168	0.0013	-0.0167	-0.0140
P_r =1.44	7.4015	0.5685	-0.0769	-0.0869	-0.0529	-0.0297
P_r =2.10	247.4983	15.4036	1.4324	0.3769	0.1830	0.0990

Table 3 Vorticity profile for different values of G_r and $G_m = 2$, M=2, $h_1 = 0.8$, $R = 1$, $\varepsilon = 0.01$,

$\delta = 1$, $M = 2$, $S_c = 0.6$, $K_0 = 1.0$

Y	0	1	2	3	4	5
G_r =0	0.6050	-0.0667	-0.0839	-0.0551	-0.0325	-0.0185
G_r =2	3.1461	0.6474	0.1168	0.0013	-0.0167	-0.0140
G_r =4	5.6820	1.3600	0.3170	0.0575	-8.7719	-0.0096

Table 4 Vorticity profile for different values of S_c and $G_m = 2$, $G_r = 2$, $h_1 = 0.8$, $R = 1$, $\varepsilon = 0.01$,

$\delta = 1$, $M = 2$, $P_r = 0.71$, $K_0 = 1.0$

Y	0	1	2	3	4	5
S_c =0.6	3.1461	0.6474	0.1168	0.0013	-0.0167	-0.0140
S_c =1.2	2.6844	0.6034	0.1557	0.0417	0.0112	0.0030
S_c =1.8	1.8334	0.5777	0.1861	0.0571	0.0169	0.0049

Table 5 Vorticity profile for different values of G_m and $P_r = 0.71$, $G_r = 2$, $h_1 = 0.8$, $R = 1$, $\varepsilon = 0.01$, $\delta = 1$, $M = 2$, $S_c = 0.6$, $K_0 = 1.0$

Y	0	1	2	3	4	5
G_m = 0	3.1627	0.7546	0.2034	0.0566	0.0159	0.0045
G_m = 2	3.1461	0.6474	0.1168	0.0013	-0.0167	-0.0140
G_m = 4	3.1362	0.5406	0.0302	-0.0541	-0.0492	-0.0325

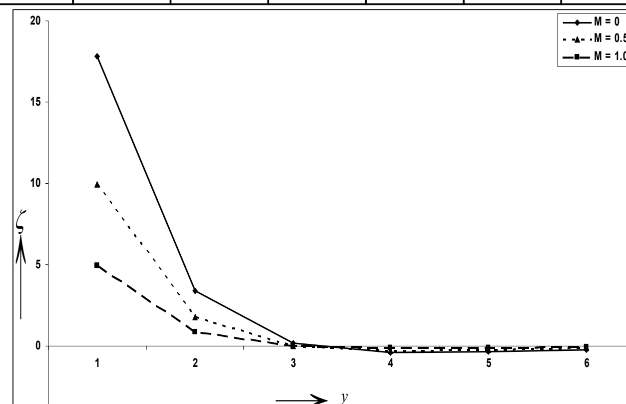

Fig. 1 Vorticity profile for different values of M

The vorticity distribution for different values of magnetic parameter (M = 0, 0.5, 1.0) with $G_m = 2$, G_r =2, M=2, $h_1 = 0.8$, $R = 1$, $\varepsilon = 0.01$, $\delta = 1$, $S_c = 0.6$, $K_0 = 1.0$ are shown in figure (1). Separately it is found that the vorticity decreases continuously with increases in y. It is observed that vorticity decreases with increasing the value of Magnetic parameter M.

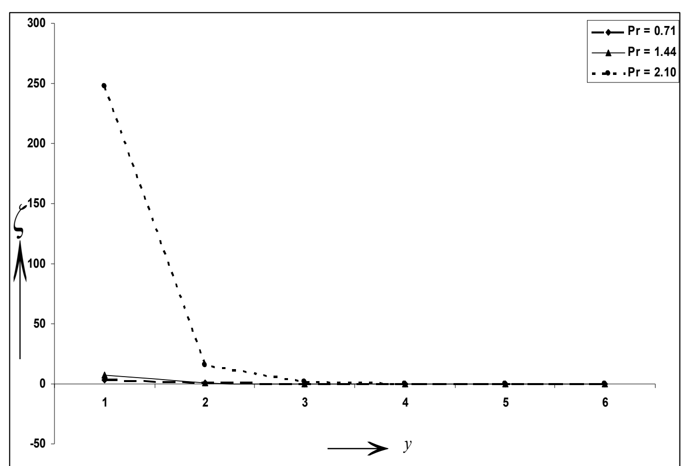

Fig. 2 Vorticity profile for different values of Pr

The vorticity distribution for different values of Prandtl number (P_r = 0.71, 1.44, 2.14) with $G_m = 2$, $G_r = 2$, $h_1 = 0.8$, $R = 1$, $\varepsilon = 0.01$, $\delta = 1$, $M = 2$, $S_c = 0.6$, $K_0 = 1.0$ are shown in figure (2). Separately it is found that the vorticity decreases continuously with increases in y. It is observed that the vorticity distribution increases due to increasing the value of Prandtl number P_r .

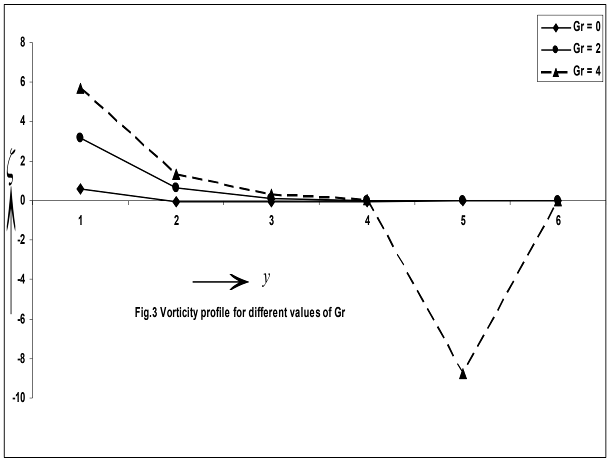

Fig.3 Vorticity profile for different values of Gr

The Vorticity distribution for different values of (G_r = 0, 2, 4) with $\mathrm{G_m} = 2, M = 2, h_1 = 0.8, R = 1,$

$\varepsilon = 0.001, S_c = 0.6, K_0 = 1.0$ are shown in figure (3). Separately it is found that the vorticity increases

continuously at y = 0 to y = 3 with increases Grashof number

G_r

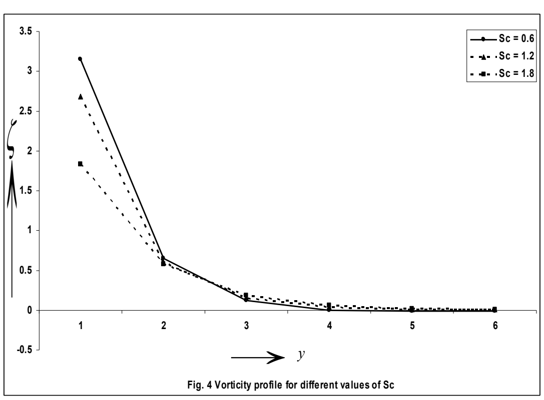

Fig. 4 Vorticity profile for different values of Sc

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The Vorticity distribution for different values of (S_c = 0.6, 1.2, 1.8), $\mathrm{G_m} = 2, M = 2, h_1 = 0.8, R = 1,$

$\varepsilon = 0.001, G_r = 2, K_0 = 1.0$ are shown in figure (4).Separately it is found that the vorticity decreases

continuously with increases y. It is observed that vorticity decreses with increasing Schmidt

number S_c .

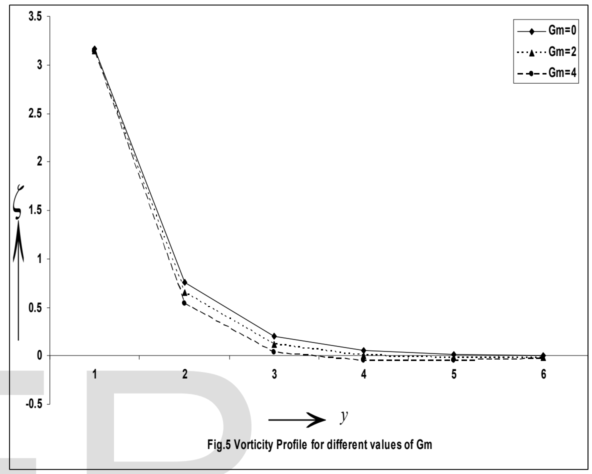

Fig.5 Vorticity Profile for different values of Gm

The Vorticity distribution for different values of (G_m = 0, 2, 4) with $\mathrm{G_r} = 2, M = 2, h_1 = 0.8, R = 1,$

$\varepsilon = 0.001, S_c = 0.6, K_0 = 1.0$ are shown in figure (5). Separately it is found that the vorticity decreases

continuously with increases in y and it is observed that vorticity distribution decreases due to

increasing the values of modified Grashof number G_m .

5. CONCLUSION

The following conclusions are drawn from the present study:

1. The vorticity decreases with increasing the values of Magnetic parameter M.
2. The vorticity increases due to increasing the values of Prandtl number P_r .
3. The vorticity increases with increasing the vales Grashof number G_r .

4 The vorticity distribution decreases with the increasing in Schmidt number S_c.

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